STATISTICAL MECHANICS

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IISER Mohali

PRE-REQUISITES : Classical Mechanics, Mathematical Methods, Quantum Mechanics

INTENDED AUDIENCE : Students in their BS-MS curriculum, BSc,/MSc. and first year PhD

COURSE OUTLINE :

The course provides a fundamental understanding of Thermodynamics and Statistical Mechanics. It starts with the fundamental concepts of thermodynamics and builds on its foundation, the principles of statisticall mechanics. It does not require a prior exposure to the topics.

ABOUT INSTRUCTOR :

Prof. Dipanjan Chakraborty completed his graduation from Presidency University (formerly Presidency College), Kolkata in 2001, followed by MSc. in Physics from IIT Kanpur. He got his PhD degree from Jadavpur University, Kolkata in 2010. Subsequently he did two post-doctoral stints in ITP, Leipzig, Germany and MPI-IS, Stuttgart, Germany. He joined the Physics department at IISER Mohali in 2013.

COURSE PLAN :

Week 1- 4:

- Introduction to Thermodynamics – the idea of macroscopic and microscopic variables with examples: a classical ideal gas and a magnetic spin system. Description of a thermodynamic system and the choice of macroscopic variables. Connection with statistical mechanics.
- Zeroth law of thermodynamics: The concept of empirical temperature arising from the zeroth law.
- First law and perpetual machines of first kind. Concept of exact and inexact differentials (for example heat and work). Similarity with the conservation of mechanical energy. Introduction of generalized forces and generalized coordinates in a thermodynamic system. The idea of response functions and their measurements.
- Second law of thermodynamics: Concepts of heat engines. Carnot engine and its efficiency. Relation with empirical temperature. Kelvin and Clausius statement of second law and their equivalence. Clausius theorem and its proof. First, second and third TdS equations.
- Extensive property of Entropy and Internal Energy. Euler theorem and Gibbs-Duhem relation from this. Illustration of Gibbs-Duhem relation for an ideal gas. Partial derivatives.
- Legendre transformations and thermodynamic free energies. Their application in real life. Introduction to Jacobians and Maxwell's relation. Use of Jacobians in deriving thermodynamic relations. Relation between specific heats.
- Stability criteria for thermodynamic systems. Concavity of entropy and convexity of internal energy. Consequence of concavity of entropy and convexity of internal energy – constraints on response functions.
- Brief introduction to theory of probability. Rules of probability. Interpretation of probability based on symmetry or frequency. Conditional probability – its interpretation in terms of sample space. Partition theorem and Bayes theorem.
- Examples of probability distribution – discrete distributions such as Binomial and Poisson. Continuous probability densities – introduction to a random variable. Normal distribution. Correlation functions. Generating functions. Central limit theorem. Example of a random walk on discrete lattice. Information and uncertainty – connection with statistical mechanics.

Week 5-8:

- Microcanonical Ensemble: Concept of classical probability density – Liouville equation, incompressible flow in phase space, meaning of ensemble average and time average of a physical quantity, ergodic theory. Equal probability and the microcanonical ensemble. Connection with thermodynamics. Counting of states – discrete case and continuous random variables.
- Canonical ensemble: Understanding what is a canonical ensemble and why we need it – connecting it with thermodynamics. Derivation of the probability density in the canonical ensemble. Fluctuations and their significance – positivity of the response functions. Examples of non-interacting systems. Equipartition theorem.
- Grand canonical ensemble. Probability density in grand canonical ensemble. Fluctuations and their significance. Generator of canonical ensemble.

Week 9-12:

- Quantum statistical mechanics: concept of averages in quantum stat. mech. Mixed and pure states and definition of density matrix. Density operators in quantum stat. mech. For different ensembles. Canonical density matrix for single particle. Wigner transformation. Examples with free particle and harmonic oscillator.
- N-particle wavefunction and canonical density matrix for N-particles. Concept of symmetric and anti-symmetric wave functions – symmetry and permutation operator. Their eigenvalues. Determination of the normalization of symmetric and anti-symmetric wave functions. Calculation of the N-particle canonical partition function for a system of ideal gas. Second virial coefficient and pair-interaction in an ideal gas.
- Grand canonical formulation of non-interacting Fermions and Bosons. Thermodynamic quantities in terms of Fermi and Bose integrals. High temperature expansion of a Fermi gas. Degenerate Fermi gas. Low temperature expansion and thermodynamic quantities at low temperature.
- Bosons: The ideal Bose gas and derivation of thermodynamic quantities in terms of Bose integrals. Bose-Einstein condensation. Specific heat of a Bose gas. Harmonically trapped Bose gas.